Equals Math: A Framework of Math Instruction for Students with Disabilities

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Overview

The necessity for a mathematics curriculum that addresses the needs of students with disabilities has been identified. Equals Math is a K-12 standards-based accessible curriculum that employs features identified in research literature on math instruction for students with disabilities. A developmental learning program in a suburban Midwestern school district conducted a year-long test of the effectiveness of the Equals curriculum. There were 72 students (ages 5-14) who completed the study. There were 21 participating teachers. Data on student progress was collected from tests given pre- and post-instruction, using the assessment protocol that is included in Equals. Teacher feedback was gathered by a survey at the end of the year. All students demonstrated progress, including students with significant disabilities. Teachers reported that the Equals curriculum was effective in helping students develop math skills and that it provided a framework for teaching students who face many challenges in learning mathematics. Not only were teachers successful in providing access to math for all their students, bur many students also exceeded their teachers' expectations in their performance.

What Research Tells Us

Since the 1997 amendments to the Individuals with Disabilities Act (IDEA), there has been a pressing need for a math curriculum for students with disabilities that provides meaningful, standards-based math instruction (Warger, 2002; Council for Exceptional Children, 2011). Jimenez (2011) suggests that it is useful to see the need for such a curriculum in the context of changing curricular perspectives over the past four decades. In the 1970s students with significant disabilities of all ages were generally exposed to adaptations of early childhood mathematics curricula. In the 1980s math instruction for these students developed around functional life skills. In the 1990s emphasis was given to self-determination and social inclusion. In the 2000s access to general academic content was grafted into the prevailing trends set in motion earlier (social inclusion, functional skill development, self-determination).

Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman (2008) conducted a meta-analysis of research on mathematics instruction for students with various disabilities. They found that the math instruction for students with significant disabilities consisted largely of computation, measurement, and money skills. Similarly, students with moderate intellectual disabilities have received math instruction focused around money skills, counting, number matching and identification, and telling time. These skills have most commonly been taught in the context of daily living activities such as shopping and following a schedule. While algebra had been part of the scope of instruction with some students with Learning Disabilities, Browder et al. found an absence of investigations of instruction in standards-based mathematical skills for students with disabilities.

Jimenez (2011) identifies four areas of difficulty that students with significant disabilities often experience with standards based math instruction: (1) understanding basic math vocabulary (reading and discussion of math concepts); (2) familiarity with basic math operations (identifying and executing strategies for solving problems and equations); (3) exposure to math (lack of previous instruction and limited real-life experience with math); and (4) challenges with memory (remembering math facts and concepts).

Ahlgrim-Delzell, Knight, Jimenez, & Agnello (2009) identified a need for models of instruction for teachers of students with significant and moderate disabilities. They called for a clearly articulated conceptual framework for what it means to teach standards-based general academic content to these students. The dearth of information available about appropriate instruction for these students leaves teachers with little guidance with regard to effective instructional strategies.

Cawley (2002) found that presenting math concepts in a way that is meaningful helps students with disabilities develop understanding of how and when to apply these concepts. He suggests that much of math instruction emphasizes "doing" and neglects "knowing". Gagnon & Maccini, (2001) found that students with disabilities come to math with poor computational and reasoning skills, and lack the ability to identify significant information in word problems. Nonetheless, with the proper supports in place, these students can experience success. Montague, Applegate, & Marquard (1993) contend that students with disabilities develop problem solving skill more effectively when they are taught how to think about problems (metacognitive processes) as well as how to "do" problems (cognitive strategies).

Maccini & Gagnon, (2000) assert that it is the deeper understanding of math that will support and serve students functionally in the real world, not surface knowledge or tricks. While it is important to identify portable, practical math tools for use in the community and at home (e.g., calculator), knowing when to use such a tool is a primary objective (Thompson & Sproule, 2000). In addition, when students are taught mathematics concepts in relationship to the system of numbers, they are better equipped to move forward with learning higher level thinking skills (Moore, 1973; Pellegrino and Goldman, 1987; Gersten, & Chard, 1999). Providing opportunities for students to work to their potential raises expectations. Higher expectations contribute to achievement (Englund, Luckner, Whaley, & Egeland, 2004; McKown, & Weinstein, 2008).

Often prerequisite skills have been embedded within a standard, as it was assumed students in general education already possessed those skills. However, students with disabilities often miss the experiences in and out of school that build those prerequisite skills due to limited opportunity, physical or cognitive access, medical needs, and/or communication deficits (Mercer, Jordan, & Miller, 1996; Mercer, Lane, Jordan, Allsopp, & Eisele, 1996; Mercer &

Mercer, 1998; Miller & Mercer, 1997). Thus proper sequencing of skills is necessary in order to address the learning needs of these students.

Students with disabilities typically need longer processing time, specialized materials, and adaptations to learn, apply, and practice math concepts; thus it often becomes necessary to break many objectives into much smaller chunks (Nadis, 1993; Miller & Koesling, 2009). It is more challenging for students with disabilities to learn multiple skills in a single lesson, as is common in general education curricula.

Equals Math Curriculum

AbleNet, Inc., an assistive technology and curriculum company for individuals with disabilities, accepted the challenge to develop a mathematics curriculum, Equals™ Mathematics. Equals was developed by drawing from general education math standards from around the country and best practice math instruction, learning strategies, and pedagogy from a wide variety of resources and research materials. These included NCTM recommendations and periodicals, publications from other sources (books, papers, journals, and periodicals) major math publishers, math instructors and special educators. In addition, action research was conducted in large and small school districts across the United States with K-12 students who have mild, moderate, and significant disabilities (Meyer, Ross-Brown, & Satterfield, 2010).

Equals instruction is organized around the five content standards identified by the National Council of Teachers of Mathematics (NCTM) which include: (1) Numbers and Operations, (2) Algebra, (3) Geometry, (4) Measurement, and (5) Data Analysis and Probability (NCTM, 2000). The curriculum incorporates the associated NCTM Process Standards which stress real-world connections, communication, multiple representations, reasoning, and problem solving in addition to knowledge of formulae and process.

Equals Mathematics was written to teach students math concepts and problem solving by using an array of tools and applying concepts to real life. The curriculum seeks to focus upon developing student understanding of when and how to use skills and concepts. Equals was designed to provide for students' unique needs with clear, simple, and singular objectives. These objectives, taken from state math standards across all regions of the United States, were written into the program to provide a sequence of skills to include the necessary prerequisites and fit the pacing and learning needs of students with disabilities, including students with the most significant needs.

The length and breadth of the lessons in Equals have been crafted to provide meaningful, connected lesson objectives in a sequence that takes full advantage of the connectivity within and between the six math content areas identified within the curriculum (numbers and operations, geometry, algebra, data analysis and probability, measurement, and attending and exploring). Equals does not have grade level boundaries, so students can continue to learn from where they left off from year to year and school to school. However, as Equals was developed with grade level standards in mind, it represents a full math curriculum, addressing skills typically taught in general education grades pre-K – 4 that are aligned to K-12 standards. The set of skills within this range are the skills students need to move forward successfully in mathematics and frequently the skills students with disabilities struggle to understand. The lessons are designed to be age-neutral so all students, no matter their age, feel they are using age-appropriate materials and can succeed.

The basic structure of each Equals lesson is consistent and systematic. Each lesson follows the same set of guiding principles with a consistent format and presentation. The components include the following best practice math instruction methodologies present in every lesson (directly related to each objective and written into the lesson in a progression of learning):

- activation of background knowledge by way of review of past related vocabulary and concept
- exploration for building background knowledge
- connection of new concept to background knowledge
- verbal communication about the math concept
- modeling written communication about the math concept
- concrete representation of math concept/vocabulary
- math vocabulary instruction
- CSA (Concrete - Semi-Concrete - Abstract) learning sequence instruction and exploration
- skill practice
- problem solving instruction
- problem solving
- math journaling
- formative assessment
- sensory math activity
- real-life problem solving connections
- workstations
- games for practice
- home letter communication and support activities

Equals CSA Methodology

Equals derives its educational foundation from the Concrete, Semi-Concrete, Abstract (CSA) pedagogy found in general education mathematics curricula (Allsopp, 1999; Allsopp, et al., 2008; Fahsl, 2007; Jordon, Miller and Mercer, 1998). This instructional approach has been demonstrated to be successful with students in elementary grades (Allsopp, 1999; Allsopp, et al., 2008; Fahsl, 2007; Harris, Miller and Mercer, 1995, Jordan, et al., 1998). CSA further draws upon brain research showing that learning is retained longer when presented in multiple formats (Sousa, 2001). Jordan et al. (1998) demonstrated the efficacy of CSA instruction with students with disabilities. Nonetheless, CSA remains largely unused beyond early grades in many school settings (Bender, 2009).

The CSA approach supports teachers with the means to provide students the opportunity to use concrete objects, images, numbers, and math symbols to help them grasp math concepts. In this way the Equals curriculum helps each student learn math in the manner that they learn best and helps them build background knowledge and a wider view of all that math entails.

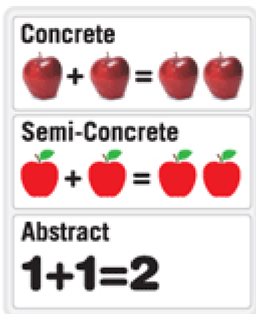

Fig. 1 Illustration of CSA

Equals Assessment

Teachers need assessments that are appropriate for learners and that provide information that can be used to identify where instruction may begin (Rulon, 2002). While traditional testing is convenient and effective for assessing the knowledge possessed by typical students, the process of assessing students with moderate and significant disabilities becomes much more difficult. Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman (2008) found that teachers lacked a means of effectively assessing skills and the effectiveness of math instruction for these students in the five mathematical components identified by the NCTM (2000).

The Equals Math Curriculum has an assessment tool. This tool is based upon the Curriculum-Based Measurement developed at the University of Minnesota (Deno, 1985). Designed to be administered to collect baseline data or to establish the entry point for instruction, the Equals assessment may be used as a pre- and post-test to monitor and document progress. This tool is divided into six subtests that correspond to the NCTM components with the addition of one component that provides visibility to pre-emerging academic skills. The six Equals assessment sub-tests are: (1) Attending & Exploring, (2) Data Analysis and Probability, (3) Algebra, (4) Geometry, (5) Measurement, and (6) Numbers and Operations. When the Equals Assessment is administered, students are assessed in each subtest. Equals includes specific instructions to adapt test questions to help students demonstrate their knowledge. Students may respond to questions in a way that best meets their needs and abilities. Adaptations to test items and adapted responses are tracked and used to adjust the Equals raw score. A formula is employed to align the adjusted raw score with a suggested starting point for Equals instruction.

The primary task in developing the Equals assessment was to identify barriers to performance to understand better what students know and what they were thinking. These barriers were viewed as cognitive, motor, and language disabilities. The Equals assessment's adapted test items are tasks with motor and expressive language performance removed, with additional support given by way of visuals and manipulatives. Thus the tasks meet students where they are in their mathematical thinking while also staying true to the math concept being assessed. In this way, full and/or incremental math knowledge can be discovered and measured.

Differentiated Instruction and Increased Expectations

Differentiation is structured within each lesson in three levels. Teachers can choose a level to match a student's individual instructional needs and then change the level it as needed to address unique needs. While the initial choice of level is based on student needs, this provides a flexible system of differentiation from lesson to lesson.

Students with severe/profound disabilities have significant challenges with language, motor, and/or cognitive skills (Jimenez, 2011). Generally, these students require assistance throughout the day for safety, to complete daily cares, and for learning. Students with severe/profound disabilities typically benefit from instruction at Level 1.

Students with moderate disabilities have similar challenges as students with severe disabilities but to a lesser degree, although a student with moderate disabilities can have significant challenges with motor and/or language skills. When supports are placed in the environment, students with moderate disabilities can usually perform independently. Students with moderate disabilities benefit from instruction at Level 2.

Students with mild disabilities have fewer challenges with cognitive skills. Language and/or motor disabilities may be present to a significant degree, but not across all three areas. Students with mild disabilities benefit from instruction at Level 3.

While teachers look to the three levels to differentiate instruction, they are meant to be flexible, as unique needs vary greatly from student to student. The levels are meant to be used as a guide for teachers to choose and adjust as they see fit.

With three levels embedded in every lesson, worksheet, workstation, and test, assumptions of ability for individual students are eliminated and expectations are raised. The principle applied here is that all of math is for every student.

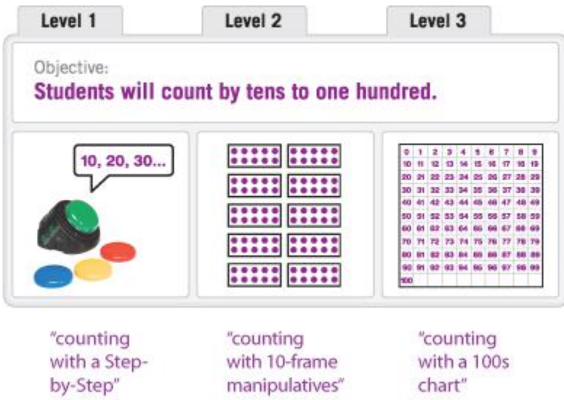

Fig. 2 Example of Equals' leveled approach to differentiation

Math Action Dictionary Differentiation Guide

In addition to the three levels, a Math Action Dictionary is provided to address access needs beyond specifics within lessons. This is a teacher resource for adaption, assistive technology, and differentiation ideas. This tool provides a global resource for differentiation based on, and organized by, actions students are asked to perform in the lessons. When a student is unable to perform an action in a typical way, the teacher can locate alternative ways in the Math Action Dictionary. Each action word entry includes 3-4 technology-based and no-tech solutions to increase engagement and ensure all students are active learners.

Research Methodology

While preliminary action research indicated that Equals was an effective Math curriculum for students with disabilities (Meyer, Ross-Brown, & Satterfield, 2010), no systematic study had been attempted. A study of the effectiveness of the Equals Mathematics curriculum was undertaken by the staff at the Developmental Learning Program from a suburban special education school district in the mid-western United States. A One-Group Pre-test Post-test research design was selected (Campbell & Stanley, 1963). The goal was to record student progress over the course of one school year. Teachers were asked to report the positive and negative aspects of the curriculum they encountered. A total of 21 teachers and 107 students with disabilities in grades K-8 (ages 5-14) were invited to take part. Of this number, data was collected on 72 students who participated for the entire year. The disabilities of the participating students included cognitive disabilities (mild, moderate, and severe), autism, multiple disabilities, and other health impairments.

Instruction in the Equals Math Curriculum was provided to participating students daily for forty (40) minute sessions in small group (3 to 8 students per group) settings. Instruction continued daily from September through May of the same school year. Baseline data was collected in September, at the outset of the school year, before the commencement of Equals instruction. Progress was assessed in April. Baseline and end-of-year progress was recorded using the

Equals assessment protocol. Both pre and post assessment data was collected by staff trained in the Equals assessment protocol, but who were unaffiliated with the students they assessed.

Teachers involved in the project participated in two two-hour training workshops on the Equals curriculum provided by AbleNet. The assessment team who would administer the pre and post testing also received training in Equals with attention to administration of the assessment itself.

Assessment

The Equals assessment was administered to the students in this study in September of 2011, at the outset of the school year, to determine each student's entry point for Equals instruction. This data also served as baseline data for this study. Students were assigned ID numbers for tracking purposes. Adjusted raw scores were associated with these ID numbers for later reference. After eight months of Equals instruction, the Equals assessment was administered again. Adapted raw scores were collected for each student ID and compared. Progress was measured by the difference in Adjusted Raw Scores between September and April.

The Equals assessment was administered by a team of educators from the school that had been trained in the Equals assessment protocol. However, these individuals did not conduct the Equals instruction for students they taught during the year. This was done to assure impartiality in the data collection process.

In addition, teachers were given a 12-question Likert survey (see Appendix 2) asking them to rate the effectiveness and ease of use of the Equals curriculum. Opportunity to make open-ended comment was provided teachers at the end of the survey questions.

Analysis

At the end of the study, individual student progress, as measured by the Equals Assessment protocol, was examined and analyzed. Calculation of actual progress for each student was made by subtracting the September adjusted raw score from the April adjusted raw score. Analysis of the progress was conducted by the instructional level in Equals, disability, age, grade, ethnicity, gender, and socio-economic status.

Analysis of Student Results

The result of the baseline (September) adjusted raw score for each student was compared to the adjusted raw scores when the assessment was administered the following April. Progress was measured by the difference in the two scores (see Appendix 1).

This analysis indicates that every one of the 72 students that completed the year demonstrated progress. No student regressed. Taken together the mean difference for all students was 28.04. This represents an average gain in math skills of 38%. Certain subgroups demonstrated even greater gains.

Students demonstrated progress at all three instructional levels. Students participating at Level 1 progressed by an average of 21.86. Level 2 students produced average gains of 24.05. Those engaged in Level 3 instruction demonstrated an average gain of 44.44.

Students made progress regardless of disability. Students with a diagnosis of autism reflected a mean difference of 32.15. Results also demonstrated noteworthy progress for students with cognitive disabilities (25.27) and multiple disabilities (21.21).

When disability group was examined in the light of instructional level, progress was evident at every level across all disabilities. Students with autism who were participating in Equals instruction at Level 1 demonstrated impressive gains (30.09). Level 2 students with autism progressed, reflecting a mean difference of 24.17. Those with autism in Level 3 instruction posted even greater progress (52.16).

Among students with cognitive disabilities, those engaged in Level 3 instruction produced average gains of 35.08. However, progress was also evident for students with cognitive disabilities participating at Level 1 (19.20) and Level 2 (25.50). Students with multiple disabilities also demonstrated progress at Level 1 (10.50) and Level 2 (22.22). No students with multiple disabilities were engaged in Level 3 instruction. Two students with other health impairments participated, demonstrating progress as well. One student was at Level 2 (23.50) and the other was at Level 3 (31.00).

When ethnicity was examined it was evident that students from all backgrounds made progress with Equals. Hispanic (34.88) and African-American (31.62) students demonstrated notable progress under Equals instruction. White (19.40) and Asian (24.00) students also showed gains. When ethnicity is examined in conjunction with instructional level, African-American students demonstrated greater progress in both Level 2 (29.56) and Level 3 (47.17) instruction than the average of all students. African American students at Level 1 evidenced progress as well (21.00). Hispanic students also produced higher gains in Level 1 (30.10) and Level 3 instruction (53.60) than those of all students taken together. Hispanic students in Level 2 produced made progress (24.93) as well. White students in Level 1 (16.33) and Level 2 (15.96) demonstrated progress, but White students in Level 3 showed above average progress (32.00).

Socio-economic status (as indicated by a student's participation in the free-lunch program) also produced marked differences. Participants in the free-lunch program presented a difference of 30.09, while their reduced lunch counterparts demonstrated a difference of 21.75, and 24.67 for students on the regular lunch program. When viewed in relation to instructional level, students with free lunch involved in level 3 instruction produced gains of 52.1.

The analysis indicated that both boys (29.42) and girls (19.45) made progress, as did students in each of the grade levels examined. The most prominent gains were demonstrated by second grade students (37.50) and seventh graders (29.46).

Test for Statistical significance

To ensure that these results were not the result of random factors, a test for statistical significance of this data was conducted by applying a two tailed t-test for two paired sample for means at the 0.05 level. This calculation produces a result known as "p-value". When the p-value calculation is less than 0.05, we can have confidence that the results are not random. This analysis found p=0.00 for all students and p<0.05 for all sub groups with the exception of sub-groups with very few participants (see Appendix 1).

Analysis of Teacher Surveys

Twenty-one teachers took part in a survey at the conclusion of the school year to assess the effectiveness of the Equals Math curriculum used with their students. Twelve questions were asked in a five-point Likert scale survey (where 1 is least effective and 5 is most effective). Opportunity to comment was then provided (see Appendix 2).

Descriptive statistics (mean and median calculations) were used to analyze the results of the survey. Ten of the twelve items received mean scores of 4.0 or higher with medians of 4 or higher. The highest rating (4.62) and greatest consensus among the teachers (all teachers gave this item a 4 or 5) related to how effective the Equals curriculum's spiraling design was for students.

Equals also received high scores on questions relating to increasing effectiveness at producing student learning across a range of math concepts (4.57), and regarding the effectiveness of the curriculum at developing content and skills (4.52). Both questions reflected strong levels of consensus and posted median scores of 5.

Several other aspects of the Equals curriculum also drew high marks: the effectiveness of the manipulatives associated with Equals (4.48), the effectiveness of the Equals assessment for placement purposes (4.39) the effectiveness of the curriculum's organization (4.38), the effectiveness of the layout of Equals lesson for lesson planning (4.33) and the effectiveness of Equals for group instruction as opposed to previous group instructional strategies (4.33). Lesson layout and chapter evaluation effectiveness each received ratings of 4.00.

From the survey results, it is clear that the participating teachers regarded the Equals curriculum as well-organized and effective. They affirmed that Equals provided a framework within which to teach math to their students. Teachers confirmed that the use of Equals produced learning beyond what had been previously expected of their students.

Teacher Comments and Observations

Teacher comments echoed the survey results. Several teachers remarked that the Equals curriculum simplified lesson planning and made them feel more productive. One teacher noted: “The planning is very simplified. It is awesome to have a curriculum that is already modified for so many levels.”

Other teachers commented upon the success experienced by their students. “The curriculum has allowed our students the opportunity to learn important concepts and skills which they wouldn’t necessarily be exposed to… The student growth over the past [school year] coupled with the students ability to articulate math concepts has been outstanding.”

Another teacher commented: “It’s wonderful to see my students’ use of the math vocabulary when they aren’t in math class.”

The majority of teacher comments, however, indicated that the systematic organization of the Equals curriculum was a major factor. “The organization of the teacher’s guide, lessons and materials is very teacher-friendly. It really makes it easy for the teachers to teach and reinforce concepts.”

One teacher noted: “the curriculum presents concepts in a manner where ideas are presented and built upon in a way which seems to make sense to our students.” Another echoed her appreciation for “the sequencing of skills that continue to build throughout the program.”

Regarding the ability of Equals to support multiple learning styles, one teacher stated: “The equals math curriculum has a variety of interactive activities included in every lesson, making it easy to reach different types of learners.”

A teacher stated that Equals was “…well organized. [There are] multiple levels and approaches to teach a variety of learners. Data driven programming means a much less subjective approach to teaching math to my students.”

Conclusions

It is clear from the results of the Equals assessment that all students in this study demonstrated progress in the Equals Math curriculum. Survey responses, comments, and observations from the teachers in this project confirm these findings.

As students on all three instructional levels demonstrated progress, the results appear to support the Equals strategy of presenting three levels of differentiation and the CSA structure within each lesson. Again, teacher survey results and comments affirm the effectiveness of this approach.

The fact that students engaged in Level 1 and Level 2 instruction made progress in the Equals curriculum in eight months is itself remarkable. The students in these levels are generally those with moderate to severe intellectual disabilities and those our schools have struggled to connect to the general education curriculum. They are often given alternative assessments instead of standardized testing in the school setting. To have a systematic math curriculum that can support effective direct instruction with measureable outcomes for these students would be a tremendous advantage.

It is also noteworthy that certain sub-groups appear to have made remarkable gains in this study. Students with autism presented elevated levels of progress. Results indicate that African–American and Hispanic as well as those from lower socio-economic backgrounds demonstrated greater gains (based upon the Equals assessment) than the average of all students taken together.

This is especially meaningful because students with disabilities from minority families and those from lower income areas often have the least exposure to real-life math. These students often have gaps in their math knowledge and lack the breadth and quality of experience as their same-age peers without disabilities. Students with autism also have challenges learning mathematics, as it is problematic for teachers to identify the gaps and limits of their understanding. It appears that the students in this study from these sub-groups, were particularly successful in the Equals curriculum and were able to demonstrate remarkable gains in math skill.

Teacher surveys and comments suggested that the structure of the Equals curriculum was simple for teachers to follow, well organized, and effective with students. The ease of use appeared to give teachers confidence. Teachers remarked that the structure provided by Equals gave them a framework for addressing skill and concept development for students who had formerly been challenging to teach mathematics.

Future Research

This study is the first to take a pre-experimental or quasi-experimental approach to examine the Equals Math curriculum. It will be instructive to see how the results of future studies might expand upon these findings. Of particular interest will be the relative progress demonstrated by Level 1 and Level 2 participants by other various sub-groups. Longer term studies of progress by students who have learned math within the Equals framework for multiple school years may also yield valuable insights.

While the Equals Assessment has been developed based upon the Curriculum-Based Measurement developed at the University of Minnesota (Deno, 1985), it would be instructive to compare the progress demonstrated by students engaged in the Equals curriculum to progress in other, more standardized frameworks. This will be challenging, given the fact that traditional testing does not provide a standardized format that is accessible for students with disabilities and that can measure incremental knowledge.

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Appendix 1

EQUALS Math Curriculum
Pre & Post Testing

Grouping	N	Sept (2011) Mean	April (2012) Mean	Mean Difference	p-Value (0.05)
All Students	72	73.42	101.46	28.04	0.000
Disabilities	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
Autism	36	76.75	108.90	32.15	0.000
Cognitive Disabilities	24	70.17	95.44	25.27	0.000
Multiple Disabilities	12	69.96	91.17	21.21	0.000
Instructional Level	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
Level 1	18	25.33	47.19	21.86	0.000
Level 2	38	76.16	100.21	24.05	0.000
Level 3	16	121.03	165.99	44.44	0.000
Grade Level	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
Grade K-1	9	53.28	80.39	27.11	0.001
Grade 2	7	45.79	83.29	37.50	0.006
Grade 3	10	66.15	95.10	28.95	0.001
Grade 4	15	57.20	82.97	25.77	0.000
Grade 5	9	79.72	108.22	28.50	0.000
Grade 6	3	145.00	166.00	21.00	0.052
Grade 7	12	90.54	120.00	29.46	0.002
Grade 8	7	104.00	127.29	23.29	0.028
Race	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
African-American	29	78.91	110.53	31.62	0.000
Hispanic	17	59.71	94.59	34.88	0.000
White	24	79.83	99.23	19.40	0.000
Asian	1	59.50	78.50	24.00	0.000
Other	1	12.50	31.50	19.00	0.000
Gender	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
Girls	10	67.45	86.90	19.45	0.000
Boys	62	74.39	103.81	29.42	0.000
Socio-Economic Status	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
Free Lunch Program	49	71.22	101.31	30.09	0.000
Reduced Lunch	8	59.94	81.69	21.75	0.0003
Regular Lunch	15	87.80	112.47	24.67	0.0002

EQUALS Math Curriculum
Pre & Post Testing
Level by Disabilities by Socio-Economic Status

Grouping	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
All Students	72	73.42	101.46	28.04	0.0000
Instructional Level	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
Level 1	18	25.33	47.19	21.86	0.0000
Level 2	38	76.16	100.21	24.05	0.0000
Level 3	16	121.03	165.99	44.44	0.0000
Disabilities	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
Autism	36	76.75	108.90	32.15	0.000
Cognitive Disabilities	24	70.17	95.44	25.27	0.000
Multiple Disabilities	12	69.96	91.17	21.21	0.000
Level 1	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
Autism	6	17.83	47.92	30.09	0.0009
Cognitive Disabilities	10	32.05	51.25	19.20	0.0001
Multiple Disabilities	2	14.25	24.75	10.50	0.2317
Level 2	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
Autism	21	77.93	102.10	24.17	0.0000
Cognitive Disabilities	8	80.13	105.63	25.50	0.0058
Multiple Disabilities	7	64.57	86.79	22.22	0.0003
Other	2	82.25	105.75	23.50	0.1718
Level 3	**N**	**Sept (2011) Mean**	**April (2012) Mean**	**Mean Difference**	**p-Value (0.05)**
Autism	9	113.28	165.44	52.16	0.0008
Cognitive Disabilities	6	120.42	155.50	35.08	0.0001
Other	1	194.50	225.50	31.00	NA

Level 1	N	Sept (2011) Mean	April (2012) Mean	Mean Difference	p-Value (0.05)
Free	11	26.64	48.68	22.05	0.000237
Reduced	4	15.75	38.75	23.00	0.008990
Regular	3	33.33	53	19.67	0.066135

Level 2	N	Sept (2011) Mean	April (2012) Mean	Mean Difference	p-Value (0.05)
Free	28	71.95	97.34	25.39	0.000000
Reduced	3	82.83	101.83	19	0.149585
Regular	7	90.14	111	20.86	0.029386

Level 3	N	Sept (2011) Mean	April (2012) Mean	Mean Difference	p-Value (0.05)
Free	10	118.25	170.35	52.1	0.000071
Reduced	1	168	193	25	NA
Regular	5	117.2	150.2	33	0.029864

Level 1	N	Sept (2011) Mean	April (2012) Mean	Mean Difference	p-Value (0.05)
African-Am.	6	34.33	55.33	21.00	0.008836
Hispanic	5	17.4	47.5	30.10	0.003541
White	6	25.08	41.42	16.33	0.003734
Other	1	12.50	31.50	19.00	NA

Level 2	N	Sept (2011) Mean	April (2012) Mean	Mean Difference	p-Value (0.05)
African-Am.	18	79.03	108.58	29.56	0.000013
Hispanic	7	61.93	86.86	24.93	0.000042
White	13	79.85	95.81	15.96	0.000570

Level 3	N	Sept (2011) Mean	April (2012) Mean	Mean Difference	p-Value (0.05)
African-Am.	6	119.08	166.25	47.17	0.008598
Hispanic	5	98.9	152.5	53.60	0.014073
White	5	145.5	177.5	32.00	0.002076

Appendix 2

Developmental Learning Program Survey

1. How effective is the lesson layout with regards to lesson planning?
Least Effective Most Effective
1 2 3 4 5

2. How effective is the lesson layout with regards to execution of the lesson?
Least Effective Most Effective
1 2 3 4 5

3. How effective is the curriculum organization?
Least Effective Most Effective
1 2 3 4 5

4. How effective is the curriculum content/ skills taught with regards to student growth?
Least Effective Most Effective
1 2 3 4 5

5. How effective is the spiraling curriculum (continually building upon skills in later lessons) to our students?
Least Effective Most Effective
1 2 3 4 5

6. How effective has the Equals program been at increasing student knowledge of a variety of math concepts?
Least Effective Most Effective
1 2 3 4 5

7. How effective has the planning and implementation of lessons in a group setting been in contrast to previous practice (individual lessons for each student)?
Least Effective Most Effective
1 2 3 4 5

8. How effective are the skill drill worksheets?
Least Effective Most Effective
1 2 3 4 5

9. How effective are the problem solving worksheets?
Least Effective Most Effective
1 2 3 4 5

10. How effective are the chapter assessments?
Least Effective Most Effective
1 2 3 4 5

11. How effective are the manipulatives in conjunction with the lesson?
Least Effective Most Effective
1 2 3 4 5

12. How effective is the students' placement test in correctly assessing students for placement?
Least Effective Most Effective
1 2 3 4 5

Please share below any comments you might have:

Appendix 3

Developmental Learning Program Survey
Results

No.	Question	Mean	Median
1.	How effective is the lesson layout with regards to lesson planning?	4.33	4
2.	How effective is the lesson layout with regards to execution of the lesson?	4.00	4
3.	How effective is the curriculum organization?	4.38	4
4.	How effective is the curriculum content/skills taught with regards to student growth?	4.52	5
5.	How effective is the spiraling curriculum (continually building upon skills in later lessons) to our students?	4.62	5
6.	How effective has the Equals program been at increasing student knowledge of a variety of math concepts?	4.57	5
7.	How effective has the planning and implementation of lessons in a group setting been in contrast to previous practice (individual lessons for each student)?	4.33	4
8.	How effective are the skill drill worksheets?	3.76	4
9.	How effective are the problem solving worksheets?	3.33	3
10.	How effective are the chapter assessments?	4.04	4
11.	How effective are the manipulatives in conjunction with the lesson?	4.48	5
12.	How effective is the students' placement test in correctly assessing students for placement?	4.39	4